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On the QCD corrections to Vainshtein's theorem for $\langle VVA \rangle$ correlator

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We point out that, contrary to existing claims of the opposite, Vainshtein's theorem on the non-renormalization of the correlator of an axial and two vector currents, is only valid in the chiral limit. When quarks, that contribute to the correlator, are massive, the QCD corrections do not vanish and the transversal and longitudinal functions are renormalized differently. We compute those corrections and study their implications for QCD effects in electroweak corrections to the muon anomalous magnetic moment.

I. INTRODUCTION

A correlator of an axial and two vector currents $\langle VVA \rangle$ is a peculiar object. Because of its relation to the anomaly of the axial current, it plays important role in understanding subtle issues in quantum field theory and in certain phenomenological applications.

One such application was recently identified in the physics of the muon anomalous magnetic moment where calculation of two-loop electroweak corrections requires the correlator $\langle VVA \rangle$, Fig.1. Thanks to Furry theorem, the vector part of the Z coupling to fermions does not contribute and the diagram Fig.1 is entirely determined by the $\langle VVA \rangle$ correlator. Because of significant interest in the physics of the muon anomalous magnetic moment, driven by recent measurements [1] of this observable at Brookhaven National Laboratory, it is not surprising that the two-loop electroweak corrections to the muon magnetic anomaly were studied thoroughly. In particular, the influence of hadronic effects on electroweak corrections to the muon anomalous magnetic moment was discussed in a number of recent papers [2, 3]. While these activities resulted in a better understanding of hadronic effects in the physics of the muon anomalous magnetic moment, they also lead to a discovery of an interesting theoretical result – a novel non-renormalization theorem for the $\langle VVA \rangle$ correlator [4] (see also [5]).

The content of this theorem can be explained as follows. In a special kinematic configuration when one of the vector currents is soft, the correlator of an axial and two vector currents is described by the longitudinal and transversal functions $w_{L,T}$. In the chiral limit, w_L does not receive QCD corrections as a consequence of the Adler-Bardeen theorem on the non-renormalization of the axial anomaly [6]. The new result, discovered by Vainshtein [4], is that in the chiral limit the transversal function is also not renormalized by perturbative QCD

effects. The proof is based on the observation that in the chiral limit $w_L = 2w_T$, to all orders in the strong coupling constant α_s . Since the Adler-Bardeen theorem protects w_L from QCD corrections, the transversal function w_T is also not renormalized.

We note that these non-renormalization theorems [4, 5] are formulated in the chiral limit. Recently, Pasechnik and Teryaev claimed [7] that these results are more general and that even for massive quarks $\mathcal{O}(\alpha_s)$ QCD corrections to the longitudinal and transversal functions are absent. This result looks somewhat peculiar; for example, it is difficult to reconcile the absence of $\mathcal{O}(\alpha_s)$ corrections with other well-known properties [2, 4] of the correlator $\langle VVA \rangle$, in particular its operator product expansion. Intrigued by that, we decided to re-calculate the QCD corrections to the $\langle VVA \rangle$ correlator in the limit when one of the vector currents is soft, allowing for non-zero quark masses. In contrast to [7], we obtain non-vanishing QCD corrections away from the chiral limit. Below the details of the calculation are described.

II. CALCULATION

Consider the correlator of an axial and two vector currents

$$T_{\mu\gamma\nu}^q = - \int d^4x d^4y e^{iqx -iky} \langle 0 | T \{ j_\mu(x) j_\gamma(y) j_\nu^5(0) \} | 0 \rangle, \quad (1)$$

where

$$j_\mu = Q_q \bar{q} \gamma_\mu q, \quad j_\mu^5 = \bar{q} \gamma_\mu \gamma_5 q \quad (2)$$

are the vector and axial currents and Q_q is the electric charge of the quark q . The pole mass of the quark is denoted by m_q in what follows.

We assume that momentum k is much smaller than

q, m_q and view the current $j_\gamma(y)$ as a source of soft photons. Then, we can write

$$\begin{aligned} T_{\mu\nu}^q &= T_{\mu\gamma\nu}^q e^\gamma(k), \\ T_{\mu\nu}^q &= i \int d^4x e^{iqx} \langle 0 | T \{ j_\mu(x) j_\nu^5(0) \} | \gamma(k) \rangle, \end{aligned} \quad (3)$$

where $e^\gamma(k)$ is the polarization vector of a photon with momentum k .

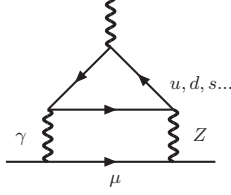


FIG. 1: A two-loop diagram that contributes to electroweak corrections to the muon anomalous magnetic moment and involves the anomalous $\langle VVA \rangle$ correlator.

We are interested in the expansion of the tensor $T_{\mu\nu}^q$ in k . Such an expansion was constructed in [2, 4]. There it was shown that, through first order in k , $T_{\mu\nu}^q$ can be written

$$\begin{aligned} T_{\mu\nu}^q &= \frac{-i}{4\pi^2} \left[w_L^q(q^2) q_\nu q^\sigma \tilde{f}_{\sigma\mu} + w_T^q(q^2) \left(-q^2 \tilde{f}_{\mu\nu} \right. \right. \\ &\quad \left. \left. + q_\mu q^\sigma \tilde{f}_{\sigma\nu} - q_\nu q^\sigma \tilde{f}_{\sigma\mu} \right) \right], \end{aligned} \quad (4)$$

where $\tilde{f}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} k^\alpha e^\beta$. The functions $w_{L,T}^q$ parametrize Lorentz structures contributing to $T_{\mu\nu}^q$, that are longitudinal (transversal) with respect to momentum q .

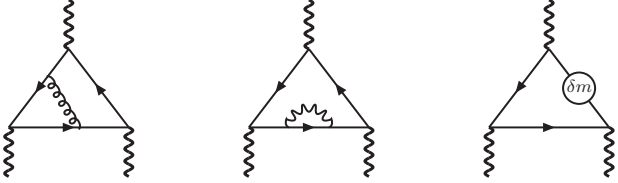


FIG. 2: Examples of two-loop diagrams that lead to $\mathcal{O}(\alpha_s)$ correction to $w_{L,T}^q$. The rightmost diagram represents an insertion of the mass counter-term.

In general, $w_{L,T}^q$ depend on q^2 and the quark mass m_q . In the one-loop approximation, these functions were computed in [8, 9]

$$\begin{aligned} w_L^{q,0} &= 2w_T^{q,0} = 2N_c Q_q^2 \int_0^1 d\xi \frac{\xi(1-\xi)}{\xi(1-\xi)Q^2 + m_q^2} \\ &= \frac{2N_c Q_q^2}{Q^2} \left(1 - \frac{2m_q^2}{\beta Q^2} \ln \frac{\beta+1}{\beta-1} \right). \end{aligned} \quad (5)$$

In Eq.(5), $N_c = 3$ is the number of colors. We also introduced the Euclidean momentum Q , $Q^2 = -q^2$ and

$\beta = \sqrt{1 + 4m_q^2/Q^2}$. In the limit $m_q = 0$, we find $w_L^{q,0} = 2w_T^{q,0} = 2N_c Q_q^2/Q^2$; as follows from the Adler-Bardeen and Vainshtein's theorems [4, 6], this is exact perturbative result in that all QCD corrections to it vanish.

In this paper, we study $\mathcal{O}(\alpha_s)$ QCD corrections to $w_{L,T}^{q,0}$ in case when the quark mass is not zero. We have to compute twelve two-loop diagrams; some examples are shown in Fig.2. Because computing $w_{L,T}^q$ beyond the one-loop approximation for arbitrary value of m_q^2/q^2 is technically challenging, we adopt a different approach. It turns out that the calculation is drastically simplified in two limiting cases $Q \ll m_q$ and $Q \gg m_q$. In addition, if sufficiently many terms in the expansion of $w_{L,T}^q$ in these limits are known, we can reconstruct the two functions completely using Padé approximation.

For both cases, $Q \ll m_q$ and $Q \gg m_q$, we employ the method of asymptotic expansions [10]. In the large-mass limit, $m_q \gg Q$, the computation is particularly simple since Taylor expansion of Feynman diagrams in Fig.2 in both k and Q suffices. Consequently, only two-loop vacuum Feynman integrals have to be evaluated in this case.

The small mass limit $m_q \ll Q$ is more complex, since Taylor expansion in m_q is insufficient. This can be seen from the one loop result which is non-analytic in the limit $m_q \rightarrow 0$. To expand the two-loop diagrams, Fig.2, in m_q , we have to consider how a loop momentum l , flowing along a line in a diagram, compares with m_q and Q . If $l \sim Q$, the propagator is Taylor expanded in m_q . In the opposite case, $l \sim m_q$, the propagator $1/((q+l)^2 - m_q^2)$ is expanded both in l and m_q while the propagator $1/(l^2 - m_q^2)$ is left unexpanded.

For each diagram, we have to consider different momenta routings and identify all possible subgraphs. Among them, there are two types of subgraphs that can be easily described. The first one corresponds to the situation when all lines in a diagram are off-shell by an amount of order Q . In this case, Taylor expanding diagram in m_q leads to a massless computation. The opposite situation occurs when both loop momenta are of order m_q . Then, a diagram is expanded in $1/Q$ and the integrals we have to deal with are the two-loop massive vacuum integrals. In intermediate cases when some of the lines in a diagram are hard $\sim Q$ and other are soft $\sim m_q$, the two-loop graph factorizes into the product of one-loop graphs.

To compute the transversal and longitudinal functions separately, we choose different momenta of the soft photon. For example, if we choose the soft momentum k in such a way that $k \propto q$, we project out the longitudinal structure function. To get rid of free Lorentz indices in that case, we contract the tensor $T_{\mu\gamma\nu}$, defined in Eq.(3), with $e^{\mu\gamma\nu\beta} q_\beta / \sqrt{-q^2}$. This procedure allows us to deal with scalar Feynman two-loop integrals for the computation of w_T^q . As we pointed out in the previous paragraph, the calculation of these integrals is simple once the limits

$m_q \ll Q$ or $m_q \gg Q$ are considered. A similar procedure is used to compute $w_{L,T}^q$.

We use dimensional regularization for the computation. For the consistent treatment of the axial current in d dimensions, we employ the procedure of Ref.[11]. The calculation is performed in an arbitrary covariant gauge; the cancellation of the gauge parameter dependence in the final result is a welcome check on the correctness of the calculation. To present the results for the longitudinal and transversal functions, we define

$$w_{L,T}^q = \frac{N_c Q_q^2}{Q^2} \left(\Delta_{L,T}^{q,(0)} + \frac{C_F \alpha_s}{\pi} \Delta_{L,T}^{q,(1)} + \mathcal{O}(\alpha_s^2) \right). \quad (6)$$

For $Q \ll m_q$, we find

$$\begin{aligned} \Delta_L^{q,(1)} &= \frac{Q^2}{3m_q^2} \left(\frac{7}{2} - \frac{617}{540} \frac{Q^2}{m_q^2} + \frac{8041}{25200} \frac{Q^4}{m_q^4} \right) + \dots, \\ \Delta_T^{q,(1)} &= \frac{Q^2}{6m_q^2} \left(3 - \frac{521}{540} \frac{Q^2}{m_q^2} + \frac{449}{1680} \frac{Q^4}{m_q^4} \right) + \dots, \end{aligned} \quad (7)$$

where ellipses stands for higher order terms in the expansion in Q^2/m_q^2 . In case $Q \gg m_q$, the functions read

$$\begin{aligned} \Delta_L^{q,(1)} &= \frac{m_q^2}{Q^2} \left(5L^2 + L + \frac{19}{6} - 8\zeta(3) \right) \\ &+ \frac{m_q^4}{Q^4} \left(-\frac{58L^2}{3} + \frac{286L}{9} + \frac{421}{27} \right) + \dots, \\ \Delta_T^{q,(1)} &= \frac{m_q^2}{Q^2} \left(\frac{5L^2}{2} - \frac{L}{2} + \frac{13}{12} - 4\zeta(3) \right) \\ &+ \frac{m_q^4}{Q^4} \left(-\frac{19L^2}{3} + \frac{145L}{9} + \frac{335}{54} \right) + \dots, \end{aligned} \quad (8)$$

where $L = \ln(Q^2/m_q^2)$, $\zeta(3)$ is the Riemann zeta-function and ellipses stands for other terms suppressed by higher power of m_q^2/Q^2 .

Several things can be pointed out in connection with these results. As expected, the QCD corrections to $w_{L,T}^q$ vanish in the limit $m_q \rightarrow 0$, in accord with the non-renormalization theorems [4, 6]. Also, the QCD corrections are not universal so that the all-orders perturbative relation $w_L^q = 2w_T^q$ is violated, once non-zero quark masses are allowed.

Using Eqs.(7,8), we can construct the entire functions $\Delta_{L,T}^{q,(1)}$ with the help of Padé approximation [12]. To this end, we compute ten terms in the expansion of $\Delta_{L,T}^{q,(1)}$ in Q^2/m_q^2 and perform Padé approximation replacing the series by rational functions. It turns out that the resulting functions do not depend on the precise form of the Padé approximation employed for their construction. For computations described below we use the [4/5] Padé approximant.

The virtue of the Padé approximation is that it allows us to continue the small- Q expansion of the functions

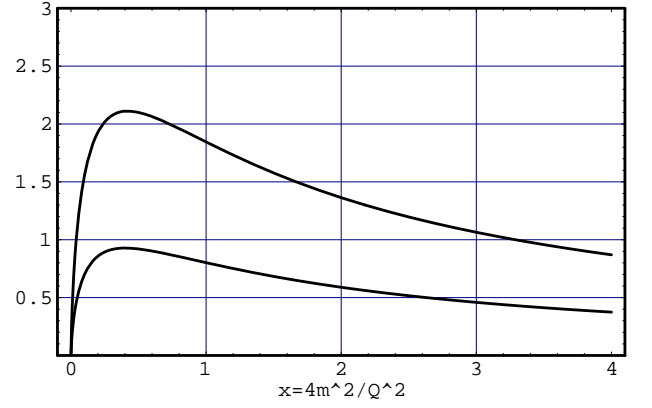


FIG. 3: The functions $\Delta_L^{q,(1)}$ (upper curve) and $\Delta_T^{q,(1)}$ (lower curve) plotted in dependence on $x = 4m_q^2/Q^2$.

$\Delta_{L,T}^{q,(1)}$ to large values of Q , where it should smoothly merge with the large- Q expansions of these functions Eq.(8). Indeed, we observe a perfect match of the Padé approximants and the large- Q^2 expansion of the functions $\Delta_{L,T}^{q,(1)}$ in the interval $10m_q^2 < Q^2 < 100m_q^2$. Hence, the Padé approximants, supplemented with large- Q^2 asymptotic, can be used to deduce $\Delta_{L,T}^{q,(1)}$ for arbitrary values of Q^2/m_q^2 .

For completeness, we present below the functions $\Delta_{L,T}^{q,(1)}$ in the form of the [4/5] Padé approximants. We introduce $z = Q^2/m^2$ and write

$$\Delta_{L,T}^{q,(1)} = \frac{\sum_{i=1}^4 a_{L,T}^{(i)} z^i}{1 + \sum_{i=1}^5 b_{L,T}^{(i)} z^i}. \quad (9)$$

The coefficients $a_{L,T}^{(i)}, b_{L,T}^{(i)}$ are given in Tables I,II. The functions $\Delta_{L,T}^{q,(1)}$ are plotted in Fig.3. It is interesting that for large range of m_q^2/Q^2 , the approximate relation $\Delta_L^{q,(1)} \approx 2.32\Delta_T^{q,(1)}$ holds with reasonable accuracy.

	L	T
$a^{(1)}$	1.1666666666	0.5000000000
$a^{(2)}$	0.3762074740	0.1620591351
$a^{(3)}$	0.03273978142	0.01418000768
$a^{(4)}$	$6.50545897 \cdot 10^{-4}$	$2.836663049 \cdot 10^{-4}$

TABLE I: Coefficients for the [4/5] Padé approximant for the longitudinal and transversal functions.

III. RELATION TO THE OPE

As we pointed out in the Introduction, the non-renormalization of the functions $w_{L,T}^q$ by perturbative

	L	T
$b^{(1)}$	0.6489185756	0.6457232084
$b^{(2)}$	0.1487376001	0.1469404864
$b^{(3)}$	0.01420626517	0.01388076895
$b^{(4)}$	$5.028490259 \cdot 10^{-4}$	$4.828123552 \cdot 10^{-4}$
$b^{(5)}$	$3.78544 \cdot 10^{-6}$	$3.527956921 \cdot 10^{-6}$

TABLE II: Coefficients for the [4/5] Padé approximant for the longitudinal and transversal functions.

QCD effects beyond the chiral limit, claimed in [7], is not possible to understand given the operator product expansion (OPE) of the tensor $T_{\mu\nu}^q$ derived in [2, 4]. In this Section, we elaborate on this statement.

Typically, the OPE is employed to estimate non-perturbative corrections to, otherwise, nearly perturbative observables. This is achieved by separating physics at different distance or momentum scales. Soft contributions, associated with non-perturbative effects, are characterized by momentum scale Λ_{QCD} whereas hard contributions are characterized by external kinematic scale $Q \gg \Lambda_{\text{QCD}}$. When the OPE is applied, soft components are identified with matrix elements of local operators while hard momenta contribute to Wilson coefficients of these operators.

Although the OPE is associated with computations beyond perturbation theory, its main idea, separation of physics at different distance scales, makes it suitable for *perturbative* computations if largely different scales are present. Hence, it is well-suited for studying longitudinal and transversal functions in the limit $Q \gg m_q$. In this case, soft scale is set by the mass of the quark m_q rather than Λ_{QCD} , but the OPE remains intact.

The OPE of the longitudinal and transversal functions reads [2, 4]

$$w_{\text{L,T}}^q = \sum_i c_{\text{L,T}}^i(\mu) \kappa_i(\mu), \quad (10)$$

where the normalization scale μ is introduced to make the separation of hard and soft modes unambiguous. In Eq.(10), the sum includes all local operators that contribute to the matrix element $\langle 0 | \mathcal{O}^i | \gamma(k) \rangle$ and behave as rank two pseudo-tensors under Lorentz transformations. The matrix elements are parametrized by

$$\langle 0 | \mathcal{O}_{\alpha\beta}^i(\mu) | \gamma(k) \rangle = -\frac{i}{4\pi} \tilde{f}_{\alpha\beta} \kappa_i(\mu), \quad (11)$$

where the dependence on the normalization point is made explicit.

From our results in the previous Section, it follows that the relation $w_{\text{L}} = 2w_{\text{T}}$ is violated once $\mathcal{O}(\alpha_s m^2/Q^2)$ contributions to these functions are computed. We would like to analyze this result from the OPE perspective. According to [2, 4], there are two operators that are required

to compute quadratic mass correction to the massless limit of the transversal and longitudinal functions. The first one is the dimension-two operator, $\mathcal{O}_{\alpha\beta}^F = \tilde{F}_{\alpha\beta}/(4\pi)$, where $\tilde{F}_{\alpha\beta} = \epsilon_{\alpha\beta\mu\nu} \partial^\mu A^\nu$; its matrix element between the vacuum and the soft photon is trivial and leads to $\kappa_F = 1$. The second operator is the dimension-three operator $\mathcal{O}_{\alpha\beta}^q = -i\bar{q}\sigma_{\alpha\beta}\gamma_5 q$. Its leading order Wilson coefficient reads [2, 4]

$$c_{\text{L}}^q(\mu) = 2c_{\text{T}}^q(\mu) = \frac{4Q_q m_q(\mu)}{Q^4}, \quad (12)$$

where $m_q(\mu)$ is the running quark mass

$$m_q(\mu) = m_q \left[1 + \frac{C_F \alpha_s}{\pi} \left(-\frac{3}{4} \ln \frac{\mu^2}{m_q^2} - 1 \right) \right]. \quad (13)$$

From the OPE of the functions $w_{\text{L,T}}^q$, Eq.(10), it follows that, in order to compute these functions through $\mathcal{O}(\alpha_s m_q^2/Q^2)$, we require the Wilson coefficient of the operator $\mathcal{O}_{\alpha\beta}^F$, the matrix element of the operator $\mathcal{O}_{\alpha\beta}^q$ and the Wilson coefficients $c_{\text{L,T}}^q$ through $\mathcal{O}(\alpha_s)$. To obtain those ingredients, two- and one-loop computations are needed. It turns out that all the matrix elements and Wilson coefficients can be extracted from the results reported in the previous Section. This happens because our calculation is based on computing contributions from different momenta regions separately. Then, soft regions in the one- and two-loop cases are identified with the matrix element of the operator $\mathcal{O}_{\alpha\beta}^q$ through $\mathcal{O}(\alpha_s)$. We have checked that identical results for this matrix element are obtained, independent of whether such identification is done in w_{L}^q or w_{T}^q . This independence is simple, yet welcome, check on the correctness of the calculation.

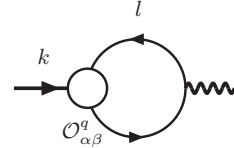


FIG. 4: The one-loop contribution to the matrix element of $\langle 0 | \mathcal{O}_{\alpha\beta}^q | \gamma(k) \rangle$. The two-loop contribution is obtained by adding all possible single gluon exchanges to this diagram.

The matrix element $\langle 0 | \mathcal{O}_{\alpha\beta}^q | \gamma \rangle$ is divergent (cf. Fig.4). To arrive at the renormalized matrix element we use the $\overline{\text{MS}}$ renormalization scheme and the fact that, under renormalization, $\mathcal{O}_{\alpha\beta}^q$ mixes with the operator $m\tilde{F}_{\alpha\beta}$. We obtain

$$\kappa_q(\mu) = N_c Q_q m_q(\mu) \left[-L_\mu + \frac{8}{3} + \frac{C_F \alpha_s(\mu)}{\pi} \left(-\frac{L_\mu^2 + L_\mu}{4} + \frac{119}{48} + \frac{\pi^2}{8} \right) \right], \quad (14)$$

where $L_\mu = \ln(\mu^2/m_q^2(\mu))$.

Using Eq.(14), we determine Wilson coefficients of the operators $\mathcal{O}_{\alpha\beta}^{F,q}$ by matching the OPE of the functions $w_{L,T}^q$, Eq.(10), to perturbative result Eq.(8). For simplicity, we give those Wilson coefficients at the normalization scale $\mu = Q$, where they do not contain any logarithms. To present the results, we introduce a short-hand notation $\bar{m}_q = m_q(Q)$ and

$$c_{L,T}^F = \frac{N_c Q_q^2}{Q^2} \tilde{c}_{L,T}^F. \quad (15)$$

Through $\mathcal{O}(\alpha_s)$, we find

$$\begin{aligned} \tilde{c}_L^F &= 2 - \frac{\bar{m}_q^2}{Q^2} \left[\frac{32}{3} - \frac{C_F \alpha_s}{\pi} \left(\frac{5}{4} - \frac{\pi^2}{2} - 8\zeta_3 \right) \right], \\ \tilde{c}_T^F &= 1 - \frac{\bar{m}_q^2}{Q^2} \left[\frac{16}{3} - \frac{C_F \alpha_s}{\pi} \left(-\frac{61}{24} - \frac{\pi^2}{4} - 4\zeta_3 \right) \right], \end{aligned} \quad (16)$$

and

$$\begin{aligned} c_L^q &= \frac{4Q_q \bar{m}_q}{Q^4}, \\ c_T^q &= \frac{2Q_q \bar{m}_q}{Q^4} \left(1 + \frac{C_F \alpha_s}{2\pi} \right). \end{aligned} \quad (17)$$

IV. THE MUON ANOMALOUS MAGNETIC MOMENT

The correlator of an axial and two vector currents in the kinematic limit when one of the vector currents is soft, appears in the physics of the muon anomalous magnetic moment, Fig.1. The correction to the muon magnetic anomaly due to these diagrams is obtained [2] upon integrating $w_{L,T}^q$ over Q^2 with the weight function

$$\Delta a_\mu = \frac{\alpha}{\pi} \frac{G_\mu m_\mu^2}{8\pi^2 \sqrt{2}} \int_{m_\mu^2}^{\infty} dQ^2 \left(w_L + \frac{m_Z^2}{m_Z^2 + Q^2} w_T \right). \quad (18)$$

In Eq.(18) we use

$$w_{L,T} = \sum_f 2I_3^f w_{L,T}^f, \quad (19)$$

and the summation index f denotes both lepton and quark contributions to the $\langle VVA \rangle$ correlator. In addition, I_3^f stands for the weak isospin of the fermion f .

In Section II we derived $w_{L,T}^q$, including $\mathcal{O}(\alpha_s)$ QCD corrections. We can use those results in Eq.(18) to obtain QCD corrections to Δa_μ . Note that, at large Q , the functions $\Delta_{L,T}^{q,(1)}$ decrease as m_q^2/Q^2 ; hence, at order $\mathcal{O}(\alpha_s)$ we can compute the integral in Eq.(18) for each quark flavor separately. We only include charm, bottom and top quarks in the computation. The up, down and strange quarks have to be treated non-perturbatively [2].

We use $m_c = 1.5$ GeV, $m_b = 4.8$ GeV and $m_t = 180$ GeV for charm, bottom and top quark masses. We also employ the leading order running of the strong coupling constant with $\alpha_s(m_c) = 0.3$. We obtain the following QCD corrections due to charm, bottom and top quarks

$$\Delta a_\mu^{q,\alpha_s} = \frac{\alpha}{\pi} \frac{G_\mu m_\mu^2}{8\pi^2 \sqrt{2}} \begin{cases} 1.80, & q = c, \\ -0.33, & q = b, \\ 0.56, & q = t. \end{cases} \quad (20)$$

The perturbative QCD correction is given by the sum of entries in Eq.(20). We obtain

$$\Delta a_\mu^{\alpha_s} = 0.55 \times 10^{-11}. \quad (21)$$

This correction is well within the error bars, assigned to hadronic uncertainties in electroweak contributions to the muon magnetic anomaly in Ref.[2]; it is negligible for phenomenology given that the current uncertainty on the muon anomalous magnetic moment is $\sim 100 \times 10^{-11}$.

V. CONCLUSION

In this paper we computed the QCD corrections to the longitudinal and transversal functions $w_{L,T}^q$ allowing for non-zero quark masses. These functions describe the anomalous $\langle VVA \rangle$ correlator in the kinematic limit when one of the vector currents is soft.

According to the Adler-Bardeen and Vainshtein's theorems, $w_{L,T}^q$ do not receive QCD corrections in the chiral $m_q = 0$ limit. Our study is motivated by the claim in Ref.[7] that the absence of QCD corrections $w_{L,T}^q$ is also valid beyond the chiral limit when quark masses are allowed.

We performed explicit calculation of the QCD corrections to $w_{L,T}^q(Q)$ for non-zero quark masses in two kinematic limits $m_q \ll Q$ and $Q \ll m_q$ and used Padé approximation to derive the approximate form of these functions valid for arbitrary relation between Q and m_q . In variance with results of Ref.[7], we found that $w_{L,T}^q$ receive perturbative corrections if $m_q \neq 0$. We argued that these QCD corrections are consistent with the well-known operator product expansion of the correlator $\langle VVA \rangle$. We studied the implications of our results for the QCD corrections to electroweak contributions to the muon anomalous magnetic moment that contain the correlator of an axial current and two vector currents, Fig.1, and found negligible effect.

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